Means

Arithmetic Mean

$$\mu = \frac{1}{n}\sum_{r=1}^{n} x_r$$

This perhaps intimidating expression simply means "sum the values up to the nth value and divide by n"

If a number μ is inserted between the numbers x_1 and x_2, such that x_1, μ, x_2 are in arithmetic progression, then μ is said to be the arithmetic mean of x_1 and x_2.

For example, if we have the numbers 3 and 4. We have two numbers, so n = 2, and summing our x values gives 7.   7/2 = 3.5

We therefore have an arithmetic progression of 3, 3.5, 4 where the difference between terms is 0.5

Geometric Mean

$$G.M. = \left(\prod x_r\right)^{\frac{1}{n}}$$

This means "multiply the values and find the nth root of the product".

If a number g is inserted between the numbers x_1 and x_2, such that x_1, g, x_2 are in geometric progression, then g is said to be the geometric mean of x_1 and x_2.

For example, if we have the numbers 3 and 4. n = 2 and the product of our x values gives 12. $\sqrt{12} = 2\sqrt{3}$

So, we have the geometric progression 3, 2sqrt3, 4. The common ratio is $\frac{2\sqrt{3}}{3}$

Harmonic Mean

$$H.M. = \frac{n}{\sum_{r=1}^{n} \frac{1}{x_r}}$$

This means "sum the reciprocal of each value and divide n by the total"

If we have 3,4,5 and 6, the sum of the reciprocals is $19/20$. 4 divided by $19/20 = 80/19$.

References

Sadler, A.J. and Thorning D.W.S (1987). *Understanding Pure Mathematics*. Oxford: Oxford University Press. p.214.
(Definitions of Geometric and Arithmetic progressions and means)
Chambers (2007). *Dictionary of Science and Technology*. 7th ed. Edinburgh: Chambers Harrap Publishers. p.563.
(Definition of Harmonic Progression & Harmonic Mean)