Kinematic Equations of Motion

$$v = u + at \quad s = \frac{(u + v)t}{2} \quad v^2 = u^2 - 2as \quad s = ut + \frac{1}{2}at^2 \quad s = vt - \frac{1}{2}at^2$$

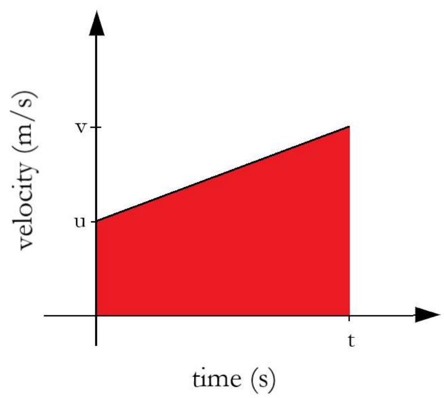
These five equations can be used to find any information about an object moving with **constant acceleration**, provided you have three pieces of information.

"u" represents initial velocity[i], "v" represents final velocity, "s" represents distance, "a" represents acceleration, and "t" represents time.

Say you wanted to find the final velocity of a car, over a span of 10 s, and you knew that the starting speed was $3ms^{-1}$ (3m/s or 3 metres per second), and the acceleration was $2ms^{-2}$ ($2m/s^2$ or 2 metres per second squared, meaning that every second the speed increases by $2ms^{-1}$). Using the equation $v = u + at$ we find that $v = 3 + 2 \times 10$. The final velocity is 23 ms^{-1}.

Proof that $v = u + at$

For any velocity-time graph, the acceleration is equal to the gradient of the graph. This graph is a straight line, which shows gradients remains constant and therefore acceleration is constant. The gradient of a line is change in y over change in x, which in this case is $\frac{v-u}{t-0}$, or $\frac{v-u}{t}$ as acceleration is equal to gradient $a = \frac{v-u}{t}$, re-arranging this gives $v = u + at$.



Proof that $s = 0.5(u + v)t$

For any velocity-time graph, it is also true that the area under the graph is equal to the distance travelled. The general equation for the area of a trapezium is $\frac{1}{2}(a + b)h$. This trapezium is 'on its side' so the height = t and a = u, b = v. Therefore $s = \frac{(u+v)t}{2}$

Proof that $v^2 + u^2 = 2as$

Re-arrange equation 1

$$v = u + at$$

to give t

$$t = \frac{v - u}{a}$$

input into equation 2

$$s = \frac{(u + v)t}{2}$$

giving

$$s = \frac{u + v}{2} \times \frac{v - u}{a}$$

re − arranging this gives

$$2as = v^2 - u^2$$

so

$$v^2 = u^2 + 2as$$

Proof that $s = ut + 0.5at^2$

Using equation 1

$$v = u + at$$

Substitute into equation 2

$$s = \frac{(u + v)t}{2}$$

Giving

$$s = \frac{(u + u + at)t}{2}$$

So

$$s = ut + \frac{1}{2}at^2$$

Proof that $s = vt - 0.5at^2$

Using equation 1

$$v = u + at$$

Re-arrange to give u

$$u = v - at$$

Substitute into

$$s = ut + \frac{1}{2}at^2$$

To give

$$s = (v - at)t + \frac{1}{2}at^2$$

So

$$s = vt - \frac{1}{2}at^2$$

See also
- Newton's Second Law

References
Attwood, G. et al. (2017). *Edexcel AS and A level Mathematics - Statistics and Mechanics - Year 1*. London: Pearson Education. pp.133-142.

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[i] Velocity is similar to speed, except that speed is a scalar quantity and velocity is a vector. A scalar quantity measures only magnitude and does not taken direction into account (if a car is travelling at 30 miles per hour, it is travelling at that speed regardless of its direction). A vector quantity takes magnitude and direction into account (if forwards is positive, a car travelling forwards would have velocity 30mph whereas backwards would be -30mph).