Differentiation

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$f'(x)$ (f prime of x) means the derivative of $f(x)$, so $f'(x)$ is a function that can be used to find the gradient of the curve $f(x)$. h is a very small value, an infinitesimally small value (you could say that $h = 0.000 \ldots 1$) this is what the expression $\lim_{h \to 0}$ tells us – h 'tends to' 0. Remember that another way of writing the derivative is $\frac{dy}{dx}$.

For example, let the function $f(x) = x^2$, and say we want to find the gradient when $x = 5$
This means that

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

Expanding $(x+h)^2$

$$f'(x) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

Cancelling the $x^2 s$

$$f'(x) = \lim_{h \to 0} \frac{2xh + h^2}{h}$$

Dividing by h

$$f'(x) = \lim_{h \to 0} 2x + h$$

Then if we let $h = 0$

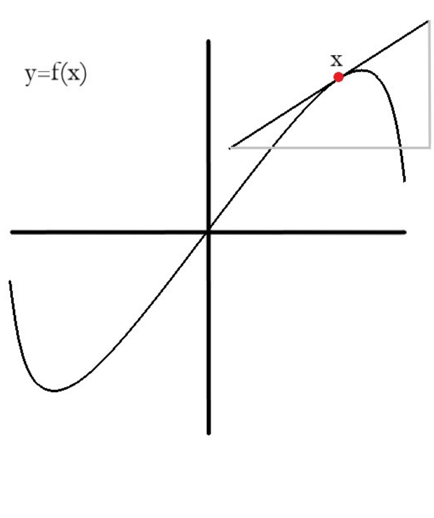
$$f'(x) = 2x$$

Then using the fact that $x = 5$

$$f'(x) = 10$$

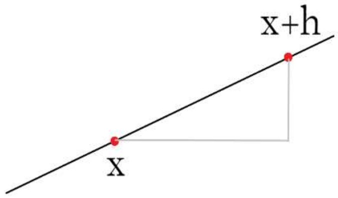
The gradient is 10.

Proof



Differentiation is the process of finding the gradient of a curve at a particular point. Another method of finding the gradient might be to draw a tangent.
We have here drawn a tangent to find the gradient at the point x, by finding the change in y and dividing that by the change in x. The problem with this method as it is not necessarily very accurate.

If, however, we know the equation of the curve we can carry out differentiation to find the exact gradient at any point on the curve.



The way we can do this is by finding the gradient between x and x+h where h is infinitesimally small.

Gradient, as mentioned, can be defined as the change in y over the change in x. So, if we use m to represent the gradient $m = \frac{y_1 - y_2}{x_1 - x_2}$. Our two x-values are x and x+h, and our y-values can be expressed as f(x) and f(x+h), because we currently do not know the function, and as stated in the previous diagram y=f(x).

Therefore

$$m = \frac{f(x+h) - f(x)}{(x+h) - x}$$

It is clear the denominator can be simplified

$$m = \frac{f(x+h) - f(x)}{h}$$

But we know that the gradient of a function is $f'(x)$

$$f'(x) = \frac{f(x+h) - f(x)}{h}$$

Finally, we add to this an explanation of what h is, that it tends to 0.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Note

It is worth noting that there is a known shortcut to this process. If you have an expression, say $f(x) = 5x^3 + 4x^2 + 3x + 2$, then you can multiply the co-efficient (the number in front of the x) by the current power and reduce the power by 1. If the number is a constant (i.e. is not multiplied by some power of x), in this example the number 2, it becomes 0 when integrated.

So in this case

$$f'(x) = 3 \times 5x^{3-1} + 2 \times 4x^{2-1} + 1 \times 3x^{1-1} + 0$$

$$f'(x) = 15x^2 + 8x + 3x^0$$

$$f'(x) = 15x^2 + 8x + 3$$

$$(x^0 = 1, \qquad x^1 = x)$$

See also

- Chain Rule
- Product Rule
- Quotient Rule
- Integration

References

Attwood, G. et al. (2017). *Edexcel AS and A level Mathematics - Pure - Year 1*. London: Pearson Education. pp.259-260