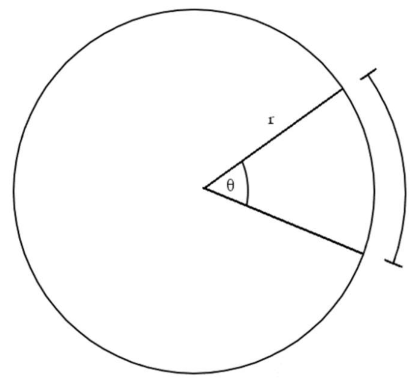
Circles

Arc length

$$l = r\theta$$

This equation is used to find the "arc length" (l) of a sector of a circle.
"r" is the radius of the circle, and θ is the angle of the sector (measured in radians[1]). A sector is a "slice" of a circle.
For example, imagine a pizza of radius 10cm cut up into eight slices. $\frac{2\pi}{8} = \frac{\pi}{4}$ so each slice, or sector, has an angle of $\frac{\pi^c}{4}$. The length of the crust is $\frac{\pi}{4} \times 10 = 7.85$ ...cm



Proof

The equation for the circumference of a circle is $c = 2\pi r$. This because π is defined as $\frac{c}{2r}$
If we need to find the arc length, this will be some fraction of the circumference. Imagine spinning around in a circle. To complete a full circle you must spin 2π radians. If you only spin a certain amount, say θ radians, we can use this information to find the proportion of the radius that you have spun round in.
Using radians, that means that this fraction can be written as $\frac{\theta}{2\pi}$
Therefore, $l = \frac{\theta}{\cancel{2\pi}} \times \cancel{2\pi} r = r\theta$

Area of a Sector

$$A = \frac{1}{2}r^2\theta$$

Proof

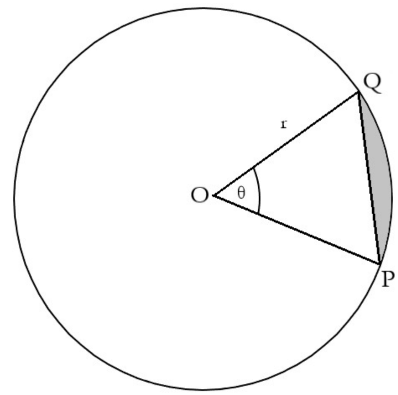
The area of a circle can be found by the equation $A = \pi r^2$.
As discussed, a sector is a part of the circle, or part of the $2\pi^c$ which makes up the circle. So the area of a sector can be written as $A = \frac{\theta}{2\pi} \times \pi r^2$
This πs cancel to make $A = \frac{\theta}{2}r^2 = \frac{1}{2}r^2\theta$

[1] There are 2π radians (or $2\pi^c$) in a full circle, so $2\pi^c = 360°$. Radians are often used to measure angles as they are considerably more useful and easier to use than degrees.

Area of a Segment

$$A = \frac{1}{2}r^2(\theta - \sin\theta)$$

A segment is shown as the grey shaded area in the diagram to the right. The area of the segment can be found by subtracting the area of the triangle POQ from the area of the sector.



We know that the area of the sector is given by

$$A_{sector} = \frac{1}{2}r^2\theta$$

Using the sine area rule for the area of POQ

$$A = \frac{1}{2}ab\sin C$$

Inputting values

$$A_{POQ} = \frac{1}{2}r \times r \times \sin\theta$$

So

$$A_{POQ} = \frac{1}{2}r^2\sin\theta$$

So the area of the segment is

$$A_{segment} = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$$

Both terms are multiplied by $\frac{1}{2}r^2$ so we can factorise it out

$$A = \frac{1}{2}r^2(\theta - \sin\theta)$$

See also

- Sine Area Rule

References

Attwood, G. et al. (2017). *Edexcel A level Mathematics - Pure Mathematics - Year 2*. London: Pearson Education. pp.118-123.